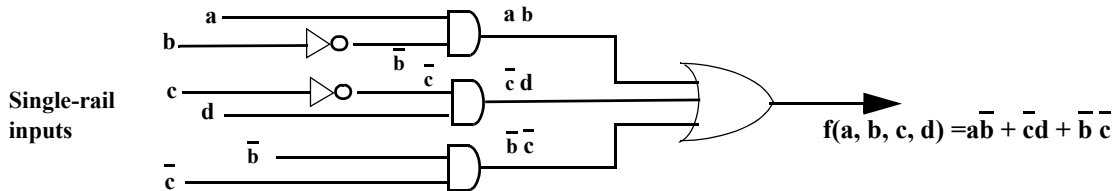


**EXAM I ANSWERS**

1) i) The simplification to obtain the minimal SOP expression :

$$\begin{aligned}
 f(a, b, c, d) &= (ac + \bar{a} + \bar{a}c + d + \bar{d}\bar{b} + \bar{a}d + \bar{a}b\bar{c}\bar{d})(\bar{a}\bar{b} + \bar{c}d + \bar{a}bcd + \bar{b}\bar{c}) \\
 &= (ac + \bar{a} + d + \bar{d}\bar{b} + \bar{a}d)(\bar{a}\bar{b} + \bar{c}d + \bar{a}bcd + \bar{b}\bar{c}) \quad k + km = k \\
 &= (ac + \bar{a} + d + \bar{b} + a)(\bar{a}\bar{b} + \bar{c}d + \bar{a}bcd + \bar{b}\bar{c}) \quad k + \bar{k}m = k + m \\
 &= (1)(\bar{a}\bar{b} + \bar{c}d + \bar{a}bcd + \bar{b}\bar{c}) \quad k + \bar{k} = 1 \ \& \ k + 1 = 1 \\
 &= \bar{a}\bar{b} + \bar{c}d + \bar{a}bcd + \bar{b}\bar{c} \quad k1 = k \\
 &= \bar{a}\bar{b} + \bar{c}d + \bar{b}\bar{c} \quad k + km = k
 \end{aligned}$$

ii) The minimal 2-level AND-OR gate network :



2) The truth table and the minterm lists :

K	K	abcd				e	w	y	z
		a	b	c	d				
0	0	0	0	0	0	0	0	0	0
1	1	0	0	0	1	0	0	0	0
-2	2	0	0	1	0	1	0	0	0
-1	3	0	0	1	1	1	0	0	0
0	4	0	1	0	0	0	0	0	0
1	5	0	1	0	1	0	0	0	0
-2	6	0	1	1	0	1	0	0	0
-1	7	0	1	1	1	1	0	0	0
0	8	1	0	0	0	0	0	0	0
1	9	1	0	0	1	0	1	1	1
-2	10	1	0	1	0	1	0	1	0
-1	11	1	0	1	1	1	0	0	1
0	12	1	1	0	0	1	1	1	1
1	13	1	1	0	1	0	0	0	0
-2	14	1	1	1	0	1	1	0	1
-1	15	1	1	1	1	1	1	1	0

The Minterm lists :

$$\begin{aligned}
 w(a, b, c, d) &= \sum m(9, 12, 14, 15) \\
 y(a, b, c, d) &= \sum m(9, 10, 12, 15) \\
 z(a, b, c, d) &= \sum m(9, 11, 12, 14)
 \end{aligned}$$

The canonical SOP expression for w(a, b, c, d) :

$$\left. \begin{array}{c}
 \frac{9}{1001} \quad \frac{12}{1100} \quad \frac{14}{1110} \quad \frac{15}{1111} \\
 \hline
 a\bar{b}\bar{c}d \quad a\bar{b}c\bar{d} \quad a\bar{b}c\bar{d} \quad a\bar{b}cd
 \end{array} \right\} z(a, b, c, d) = a\bar{b}\bar{c}d + a\bar{b}c\bar{d} + a\bar{b}c\bar{d} + a\bar{b}cd$$

The canonical SOP expression for y(a, b, c, d) :

$$\left. \begin{array}{c}
 \frac{10}{1010} \\
 \hline
 a\bar{b}c\bar{d}
 \end{array} \right\} z(a, b, c, d) = a\bar{b}\bar{c}d + a\bar{b}c\bar{d} + a\bar{b}c\bar{d} + a\bar{b}cd$$

The canonical SOP expression for z(a, b, c, d) :

$$\left. \begin{array}{c}
 \frac{11}{1011} \\
 \hline
 a\bar{b}cd
 \end{array} \right\} z(a, b, c, d) = a\bar{b}\bar{c}d + a\bar{b}c\bar{d} + a\bar{b}cd + a\bar{b}c\bar{d}$$

Since input K has two bits and output R has 3 bits, we sign extend input K by one bit. The new leftmost bit of K is “e” and its value is shown on the truth table above.

Note that based on the truth table, we can state that input “a” is the enable input that forces the outputs to be zero when it is 1. Therefore, it is an active-high input.

3) Since the first number is Hex coded, first we replace its Hex coded digits with bits. Then, we perform the addition in the 2’s Complement Binary system :

$$\begin{array}{r}
 \text{(A 5)}_{\text{Hex}} \\
 + \quad 1110 \ 1110 \\
 \hline
 ?
 \end{array}
 \left. \vphantom{\begin{array}{r} \text{(A 5)}_{\text{Hex}} \\ + \quad 1110 \ 1110 \\ \hline ? \end{array}} \right\}
 \text{(A 5)}_{\text{Hex}} = \underbrace{1010}_A \ \underbrace{0101}_5
 \left. \vphantom{\text{(A 5)}_{\text{Hex}} = \underbrace{1010}_A \ \underbrace{0101}_5} \right\}
 \begin{array}{r}
 \phantom{+} \quad 1010 \ 0101 \quad \leftarrow 0 \\
 + \quad 1110 \ 1110 \\
 \hline
 1 \leftarrow 1001 \ 0011 \quad \leftarrow 1 \\
 \text{c}_{\text{out}}
 \end{array}$$

We added two negative numbers and the result is negative. Therefore, there is **no** overflow. That is, the result is correct ! We can convert the result to decimal. Since the result is **negative**, we have to make it positive first :

$$\overline{1001 \ 0011}^2 = \underbrace{0110 \ 1101}_{7 \ 6 \ 5 \ 4 \ 3 \ 2 \ 1 \ 0} \left. \vphantom{\overline{1001 \ 0011}^2 = \underbrace{0110 \ 1101}_{7 \ 6 \ 5 \ 4 \ 3 \ 2 \ 1 \ 0}} \right\} 2^6 + 2^5 + 2^3 + 2^2 + 2^0 = 64 + 32 + 8 + 4 + 1 = (109)_{10}$$

$\Rightarrow (-109)_{10}$

4)

RD	Displays Before Play PD3 PD2 PD1 PD0	Displays After Play PD3 PD2 PD1 PD0	D/A	The Adjacency	Points Earned (Decimal)	Machine player plays again ?
5	A 5 5 2	A 5 5 (5)	D	2	20	Y
4	8 4 4 C	8 4 4 (4)	D	2	16	Y
7	E 7 7 E	E 7 7 (7)	D	2	28	Y
0	F F F C	F F (F) C	A	2	60	Y
1	7 1 7 1	7 1 (8) 1	A	0	72	N

The machine player strategy does **not** check for code digits and so misses to earn code reward points when the random digit is 4 and 0 above. On the other hand, by chance it earns code reward points when the random digit is 1.