

HOMework IV

DUE : November 11, 2009

READ :

- Related portions of Chapters 2, 3, 4 and Appendices E and H of the Hennessy book
- Related portions of Chapter 1, 4 and 6 of the Jordan book

ASSIGNMENT: There are two problems.

Solve all homework and exam problems as shown in class and past exam solutions.

1) Draw three different interconnection networks with 64 nodes :

- ⇔ A *cube-shaped* three-dimensional torus network,
- ⇔ A hypercube network and
- ⇔ A cube-connected-cycles (CCC) network.

Indicate, by explaining :

- ⇔ The degree,
- ⇔ The diameter,
- ⇔ The total number of links,

of each network,

- ⇔ For the given size of the networks and
- ⇔ For any size.

For the three networks, you need **not** show all the links and node numbers. But, show enough to give an idea about the pattern of link connections and node numbering.

2) Draw a Benes network with 8 nodes on each side.

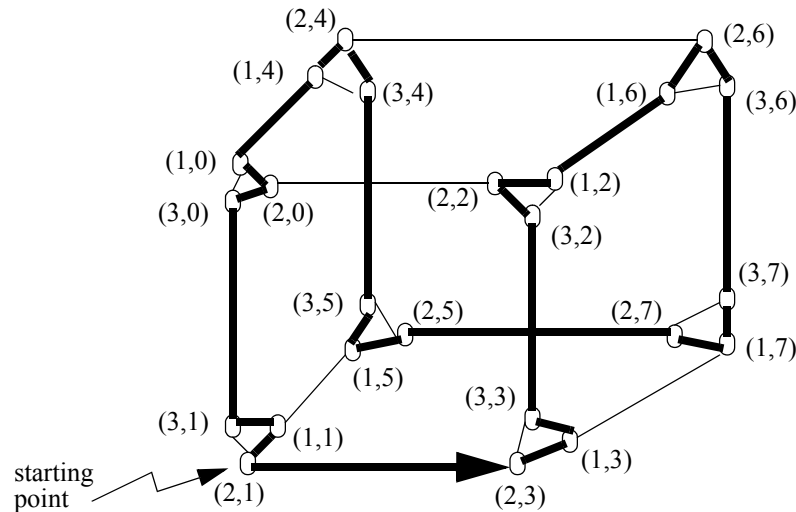
⇔ Describe its unique features and show how messages would travel from one side to the other.

RELEVANT QUESTIONS AND ANSWERS

Q1) On a 3-CCC, embed the largest ring possible. That is, form the longest path possible that starts at node “a” and ends at node “a,” forming a circle (ring). To show the embedded ring, number the nodes of the 3-CCC and write down the order of the nodes traversed when the embedded ring is traversed.

A1) The 3-CCC has 24 nodes. Each node has a unique address : (i, j) where “j” indicates which corner of the 3-CCC the node resides at and “i” indicates which node of the corner.

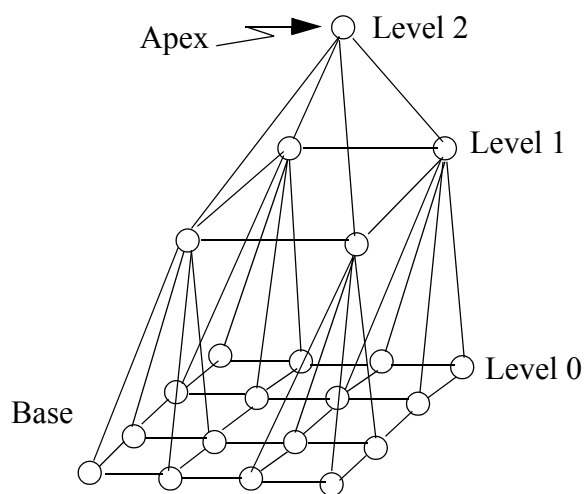
To form a 3-CCC, we connect node (i,j) to node (i,m) if and only if “m” is the result of inverting the “i”th bit (from left, starting at 1) of the binary representation of “j.” For example, for node (1,3), the connection to the next corner is to node (1,7). Because, “i” is 1 and “j” is 3 (or 011). “m” is 7 (111) since we invert bit one from left of “j.” Below is the 3-CCC with the ring embedded :



The ring includes all the nodes of the 3-CCC and can be traversed in many ways : with different starting points and different paths. All of these long rings have the same length : 24 links. This ring is shown by thick lines in the above picture. Here is one ring starting at (2,1) :

(2,1)-(2,3)-(1,3)-(3,3)-(3,2)-(2,2)-(1,2)-(1,6)-(2,6)-(3,6)-(3,7)-(1,7)-(2,7)-(2,5)-(1,5)-(3,5)-(3,4)-(2,4)-(1,4)-(1,0)-(2,0)-(3,0)-(3,1)-(1,1)-(2,1).

Q2) Consider the following static direct interconnection network :



This is a pyramid network of size k^2 or size 16. Make observations on the network, including

- (i) the number of nodes,
- (ii) the degree,
- (iii) the diameter

and **others** as a function of **k** based on the discussion of interconnection networks in class.

Note that in the pyramid network, all, except base nodes, have four children each, on the level below.

A2) A pyramid network combines the advantages of mesh and tree networks and physically “puts them together.” A k^2 -size pyramid is a 4-ary rooted tree and has $(\log_2 k + 1)$ levels. For this problem, there are three levels. Thus, k is 4. Another way to calculate k is that at the base, there are 16 nodes, where $k^2 = 16$. Therefore, k is 4.

Since the number of base nodes can be 4 ($k = 2$), 16 ($k = 4$), 64 ($k = 8$), etc., k is a power of 2.

We observe that each node has 4 children except the ones on the base level. A size k^2 pyramid has $((4k^2 - 1)/3)$ nodes. Then, N , the number of nodes, is :

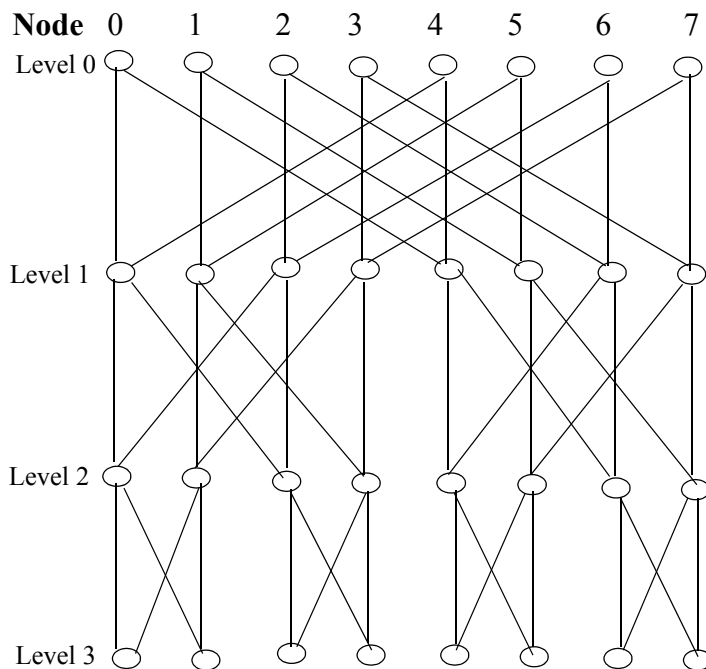
$$((4 \cdot 4^2 - 1)/3) = ((64 - 1)/3) = 63/3 = 21$$

For this problem, the intermediate level nodes have the largest degree : 7, which is 4 for the children, 2 for the mesh neighbors on the intermediate level and 1 for the parent. But, for pyramid networks of $k > 4$, every interior node has a degree of 9 : 4 for mesh neighbors on its level, 4 for children and 1 for the parent. Then, the degree of the pyramid network is 9.

The diameter of the network is $2\log_2 k$ for any k . This is the case when a base node communicates with another base node on the other side.

This network provides alternative paths between nodes, making dynamic routing possible, increasing fault tolerance.

Q3) Consider the following static direct interconnection network :



This is a network of size $(k + 1)$ where $(k + 1)$ indicates the number of levels.

Determine

- the number of nodes,
- the degree,
- the diameter

as a function of k .

Make also other observations based on the discussion of interconnection networks in class.

A3) This is a butterfly network of size $(k + 1)$ where $(k + 1)$ indicates the number of levels.

Each node has a unique id : (i, j) . A node with an id number (i, j) is on level “ i ” and node “ j ” on that level. Then, node (i, j) is connected to two nodes on level $(i - 1)$: $(i - 1, j)$ and $(i - 1, m)$

where “m” is the number obtained from the inversion of the i^{th} bit (from left, starting counting at 1) of “j.”

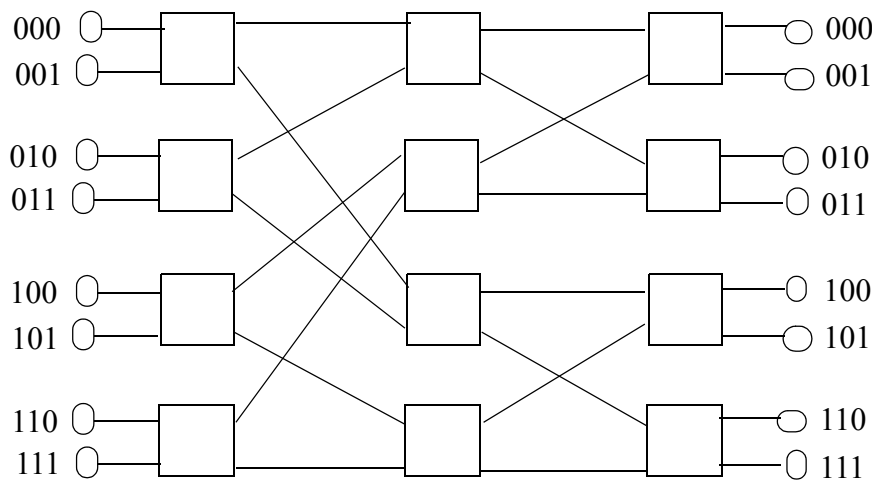
If node (i,j) is connected to node (i-1,m) then node (i,m) is connected to node (i-1,j), forming a “**butterfly**.”

Each level has 2^k nodes. Then, the butterfly network has $(k+1)*2^k$ nodes.

Each intermediate level node has 4 links while the nodes on the two ends have 2. Then, the degree of the network is 4 and constant for any size. Note that if k is 1, obviously, there is no intermediate level, so the degree is 2.

The diameter is $2*k$ which is the case if two nodes on level 0 (or level k) want to communicate. We have to move down to level k (k links) and then up to the destination by traveling on k more links.

Q4) Consider the following interconnection network :



Make observations on the network, including (i) its kind (i.e. static/dynamic, direct/indirect, etc.), (ii) the routing, (iii) blocking/nonblocking properties, (iv) the interstage connection scheme, (v) the number of stages and (vi) the number of switches, the last two of which must be in relation to the number of nodes.

A4) The network is indirect and dynamic where the switches connect the nodes on the two sides.

The number of stages depends on the number of nodes on a side such that from left to right between two successive stages we interconnect N ports, then N/2 ports then N/4 ports until we have 4 ports left. Then, we stop : $\log_2 N$ stages needed

The number of 2x2 switches = number of stages * (N/2)

The interstage connection pattern between k ports is that node “i” on the left side is connected to port “j” on the right side where “j” is the “rotate right” result on the bit pattern of node “i.”

The routing is done by inspecting the destination address, starting at the leftmost bit for the leftmost stage :

- 0 means take the upper port
- 1 means take the lower port.

The return address gradually replaces the destination address completely : we replace the inspected bit of the destination address with a

- 0 if it came from the upper port
- 1 if it came from the lower port

There are no multiple paths between right and left sides : the interconnection network is blocking.

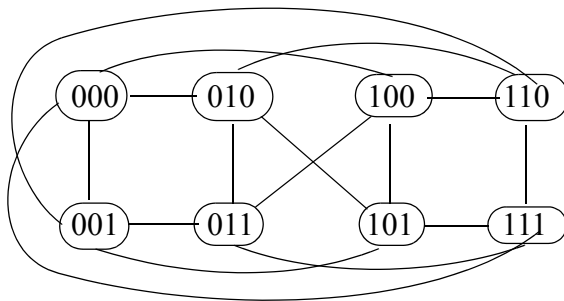
Q5) Construct a static direct interconnection network that has the following properties:

- each node has an id number represented by “k” bits, and
- two nodes are connected to each other if they differ in one bit position **or** in k bit positions.

Draw the network for k = 3. Indicate (i) the number of nodes in the network, (ii) the degree, (iii) the diameter and (iv) the number of links. Explain how you obtain these numbers.

Then, for any “k,” explain, (i) the number of nodes in the network, (ii) the degree, (iii) the diameter and (iv) the number of links. What name would you give to this interconnection network ? Why?

A5) The network for k = 3 is below :



For k = 3 =>

- The number of nodes = $2^3 = 8$
- The degree = 4
- The diameter = 2
- The number of links = 16

For any k =>

The number of nodes is 2^k

The degree is “k+1” since each node has k different neighbors for “k” 1-bit different positions and one more neighbor whose id number differs in k bit positions.

The diameter is “k-1” since for any node, the farthest node would be the node with k different bits if there was no extra link to the node with k different bits. Therefore, the longest distance is to a node whose id number differ in k-1 bit positions.

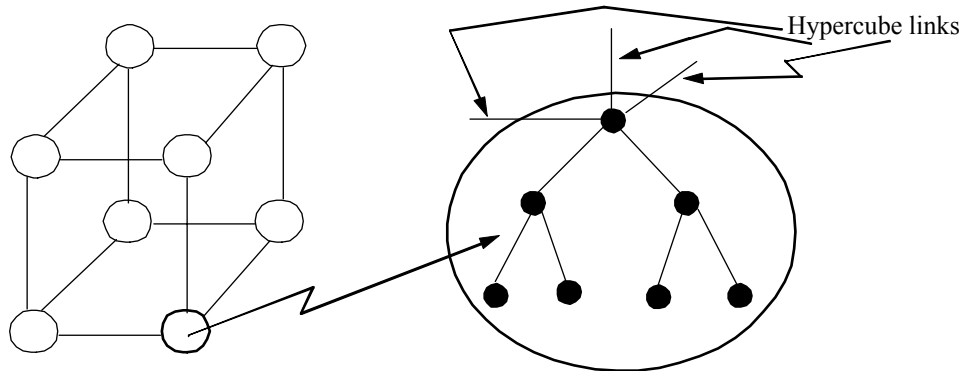
The number of links :

$$\frac{k \times 2^k}{2} + \frac{2^k}{2} = 2^{k-1} \times k + 2^{k-1} = 2^{k-1} \times (k + 1)$$

1-bit connections
k-bit connections

This network is a modified hypercube since compared with the hypercube, the only major difference is that each node receives one additional link to the farthest node (that differs in k bits).

Q6) Consider the static direct interconnection network below that is a combination of the hypercube and balanced binary tree. In this interconnection network, a hypercube of dimension “k” has a k-level balanced binary tree at each corner. In the interconnection network below “k” is 3 :



On the above interconnection network of “k” = 3, **make** observations, including (i) the number of nodes, (ii) the degree, (iii) the diameter, (iv) the total number of links and others, based on the discussion of interconnection networks in class.

Then, for any “k,” **explain**, (i) the number of nodes, (ii) the degree, (iii) the diameter and (iv) the total number of links as done in class.

A6) For the given network of size $k = 3$:

(i) The degree = 5, since the root node at each corner has five neighbors.

(ii) The diameter = 7, since it is the distance between the leaf nodes of two most distant corners where there are three links between the corners as it is a hypercube-based network, two links from a leaf node up to its root and two links from the root node down to the leaf node.

(iii) The number of nodes = 56, since each tree has seven nodes and there are eight trees.

(iv) The number of links = 60, since each tree has six links and so $8 * 6 = 48$ tree links, plus 12 hypercube links, totaling to 60 links.

For a network of size k :

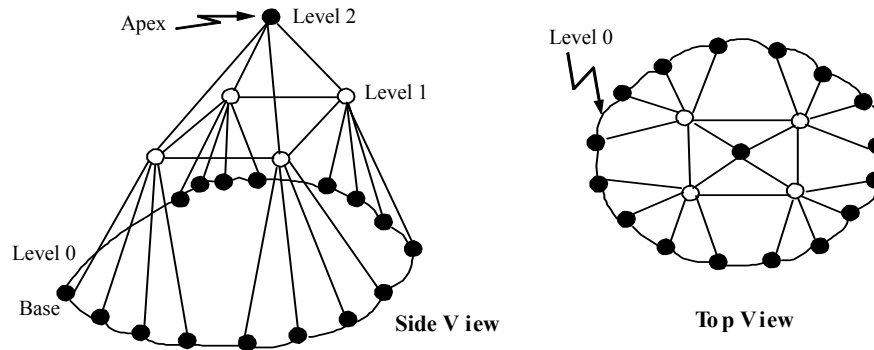
(i) The degree = $k + 2$, since each root has two children plus k hypercube neighbors.

(ii) The diameter = $k + 2(k - 1) = 3k - 2$, since k links are traversed on k -dimensional hypercube, plus $(k - 1)$ links are traversed up in one tree and down in another tree.

(iii) The number of nodes = $2^k * ((2^k) - 1)$, since there are $(2^k - 1)$ nodes in each tree and there are 2^k trees (corners).

(iv) The number of links = $2^k(2^k - 2) + k * 2^{(k-1)}$, since there are 2^k trees (corners) each with $(2^k - 2)$ links plus there are $(k * 2^{(k-1)})$ hypercube links.

Q7) Consider the following static direct interconnection network shown from the side and top :



From the side, this is a cone-shaped interconnection network of size k^2 or 16. From the top, the interconnection network looks like a hierarchy of rings of size k^2 or 16. On the interconnection network **above**, make observations, including (i) the number of nodes, (ii) the degree, (iii) the diameter, (iv) the total number of links and others, based on the discussion of interconnection networks in class.

For any “k,” **explain**, (i) the number of nodes, (ii) the degree, (iii) the diameter and (iv) the total number of links as done in class. In addition to the “k” value above, determine two other values of “k.”

A7) For the given network of size $k^2 = 16$:

- (i) The degree = 7 : Level 1 nodes have four children, two Level-1 neighbors and a parent.
- (ii) The diameter = 4 : it is the distance to travel from a Level-0 node to the most distant Level-0 node, taking two links up and two links down.
- (iii) The number of nodes = 21 : 16 on Level 0 plus four on Level 1 and 1 on Level 2.
- (iv) The number of links = 40 : 16 Level-0 links, 4 Level-1 links, 16 links from Level 1 to Level 0 and four links from Level 2 to Level 1.

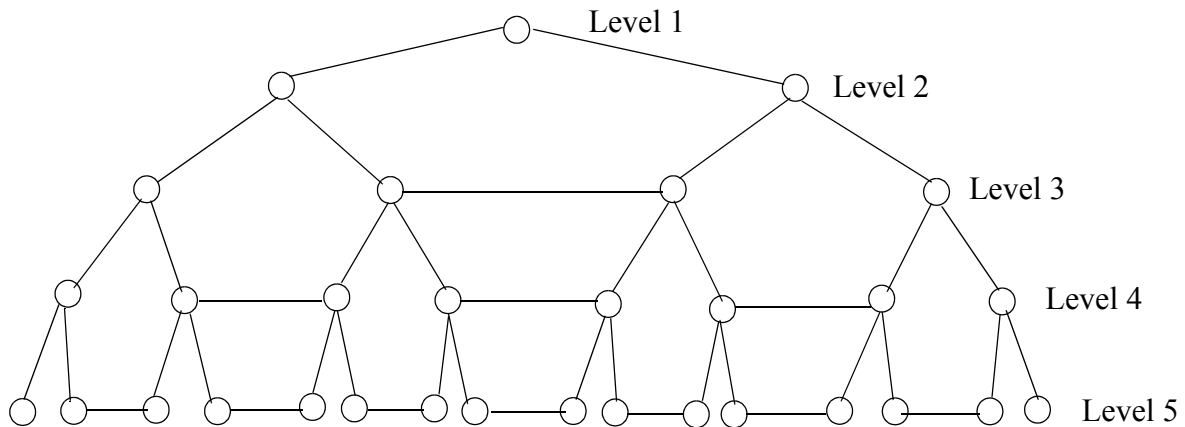
For the given interconnection network of $k^2 = 16 \Rightarrow k = 4$. The k value determines the number of base nodes which is a power of 4 as we form 4-ary rooted trees. Then, if the base has 1 node, $k = 1$. If the base has 64 nodes, $k = 8$. For 256 nodes, $k = 16$, and so on. There are alternative paths between nodes, making dynamic routing possible, increasing the availability of the network.

For a network of size k :

- (i) The degree = 7 : an intermediate node has four children, two neighbors on its level and a parent.
- (ii) The diameter = $2\log_2 k$: we climb up and down trees and their height is $\log_2 k$ each.
- (iii) The number of nodes = $(4k^2 - 1)/3$: every node except base nodes has four children.

(iv) The number of Links = $\underbrace{\left(\sum_{j=1}^{\log_2 k} 4^j \right)}_{\text{Ring links}} + \underbrace{\left(\frac{4k^2 - 1}{3} - k^2 \right) \times 4}_{\text{Tree links}} = \frac{8}{3}(k^2 - 1)$

Q8) Below, a static direct interconnection network is shown for **5 levels** or **height 4** :



On a level, two nodes are connected to each other if they do not have the **same** parent.

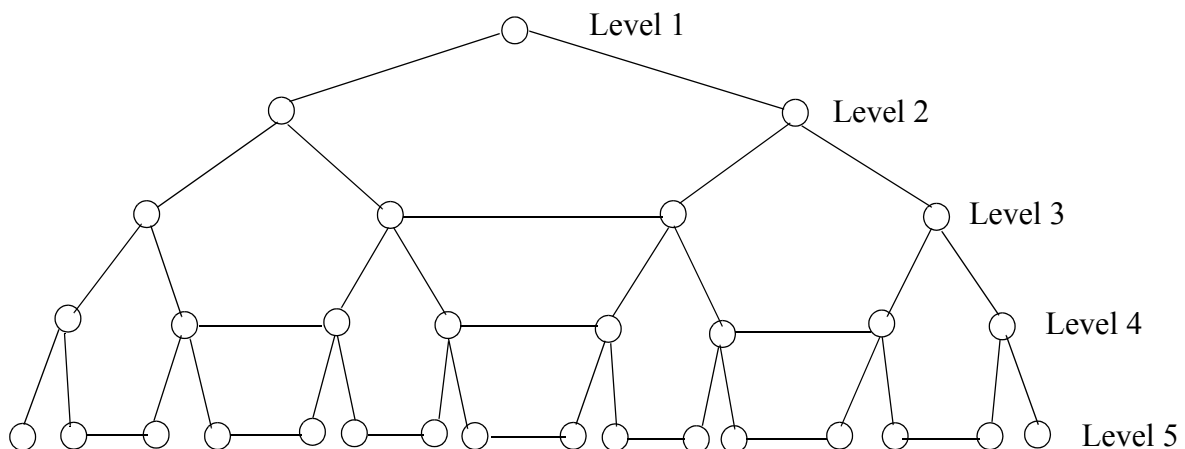
a) On the above interconnection network where level $k = 5$ or height $h = 4$, make observations, including :

- (i) the degree,
- (ii) the diameter,
- (iii) the total number of nodes,
- (iv) the total number of links **and**
- (v) **others**, based on the discussion of interconnection networks in class.

b) Then, for any “ k ” larger than 2 (or any “ h ” larger than 1), **explain** :

- (i) the degree,
- (ii) the diameter,
- (iii) the total number of nodes,
- (iv) the total number of links **and**
- (v) **others**, as done in class.

A8) The interconnection network where $k = 5$:



k = 5 :

Degree : 4, since certain inner nodes have connections to two children, a parent and a sibling : 4 links.

Diameter : 8, since the leftmost leaf has 8 links to traverse to reach the rightmost leaf.

Number of nodes : 31

Number of links : 41

For the given k values, there are multiple paths between nodes which can improve the tolerance to link and node failures. Also, on the lower levels, there is localized communication that can help certain applications.

For any “k,” larger than 2 :

Degree : 3 if k = 3 and 4 if k > 3 since inner nodes have four links.

Diameter : the distance between two nodes on the far left and right : 2 * (k - 1)

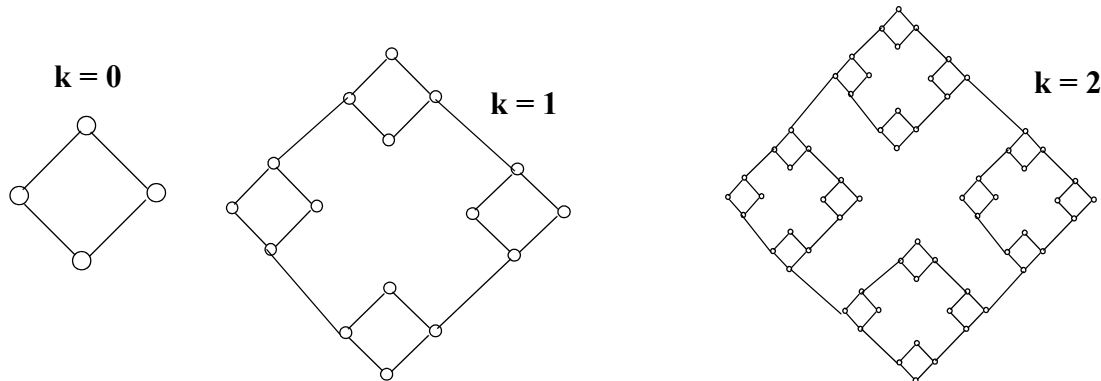
Number of nodes : 2^k - 1 since the new links besides the tree links help localized communication.

Number of links : each time the network expands, we add (2^(k-2) - 1) links besides the balanced binary tree links :

$$(2^k - 1) + \sum_{i=1}^{k-2} ((2^i) - 1) = (2^k - 2) + (2^{k-1} - k)$$

Other observations : All nodes are provided with multiple paths without increasing the number of links much, providing fault tolerance in case node and link failures.

Q9) Consider the following static direct interconnection network shown for three different sizes of “k” :

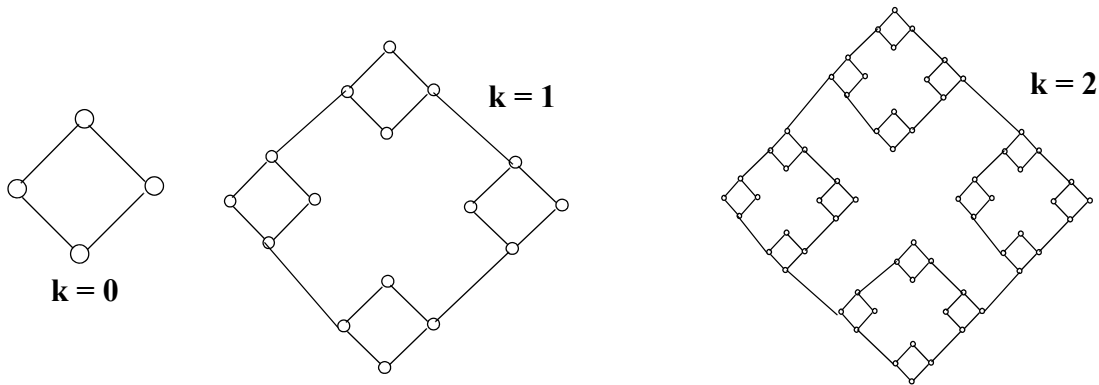


In its smallest size where k = 0, there are four nodes forming a square. The next larger size is when k = 1 : Each node of the (k=0) network is replaced by a replica of the (k=0) network. This continues indefinitely as k becomes larger.

On the **three** interconnection networks above, where k is 0, 1 and 2, make observations, including (i) the degree, (ii) the diameter, (iii) the total number of nodes, (iv) the total number of links and others, based on the discussion of interconnection networks in class.

Then, for any “k” larger than 2, **explain**, the degree, the diameter, the total number of nodes and the total number of links as done in class.

A9) The interconnection networks are given for three different values of k :



k = 0 :

Degree : 2, since all the nodes have two links.
 Diameter : 2, since the most distant nodes are two links apart.
 Number of nodes : 4
 Number of links : 4

k = 1 :

Degree : 3, since inner nodes have three links.
 Diameter : 6, since the two nodes on the far left and far right have the distance equal to six.
 Number of nodes : 16
 Number of links : 20

k = 2 :

Degree : 3, since inner nodes have three links.
 Diameter : 14, since the two nodes on the far left and far right have the distance equal to 14.
 Number of nodes : 64
 Number of links : 84

For the given k values, there are multiple paths between nodes which can improve the tolerance to link and node failures.

For any “k,” larger than 2 :

Degree : 3 since inner nodes have three links.

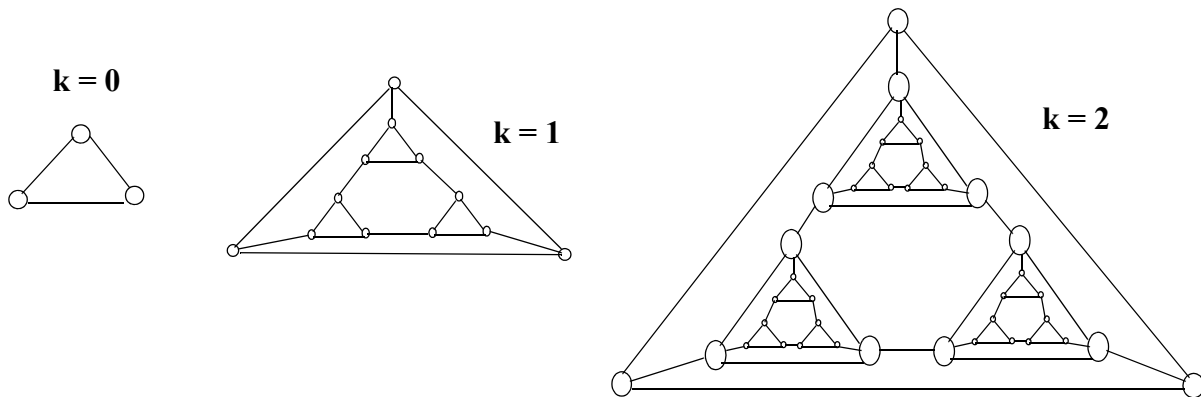
Diameter : the distance between two nodes on the far left and right : $2 * ((2^{(k+1)})-1)$ or : $\sum_{i=0}^k (2^{(i+1)})$

Number of nodes : $4^{(k+1)}$

Number of links : each time the network expands, we add $4^{(i+1)}$ links : $\sum_{i=0}^k (4^{(i+1)})$

The inner nodes are provided with multiple paths. But, outermost nodes may be the bottleneck when the network is scaled up.

Q10) Below, a static direct interconnection network is shown for three different sizes of “k” :

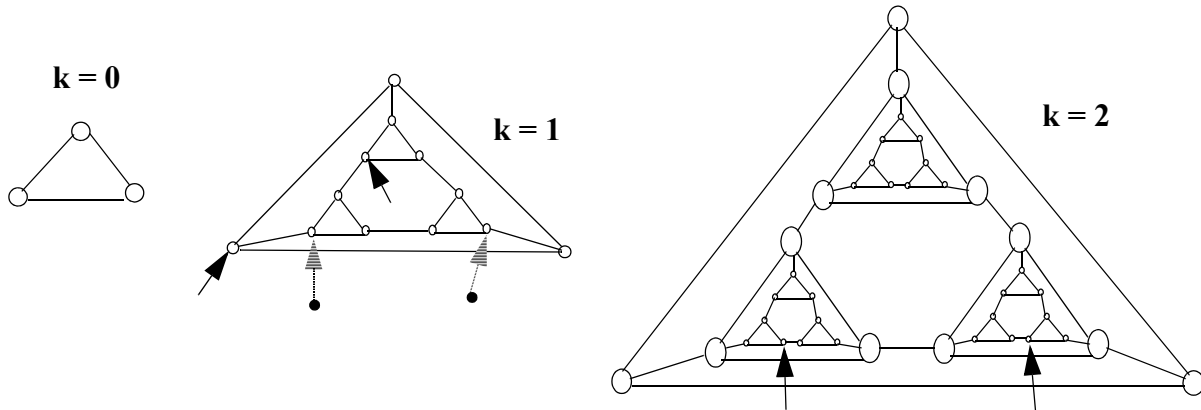


In its smallest size where $k = 0$, there are three nodes forming a **triangle**. The next larger size is when $k = 1$: Three new triangles are inserted into the $k=0$ triangle. This continues indefinitely as k becomes larger.

On the **three** interconnection networks above, where k is 0, 1 and 2, make observations, including (i) the degree, (ii) the diameter, (iii) the total number of nodes, (iv) the total number of links **and others**, based on the discussion of interconnection networks in class.

Then, for any “k” larger than 2, **explain**, the degree, the diameter, the total number of nodes and the total number of links as done in class.

A10) The interconnection networks are given for three different values of k :



$k = 0$:

Degree : 2, since all the nodes have two links.

Diameter : 1, since the most distant node is one link apart.

Number of nodes : 3

Number of links : 3

$k = 1$:

Degree : 3, since inner nodes have three links.

Diameter : 3, since most distant nodes are on the inner triangles or one on the outside triangle and one on the inside far triangle as pointed in the figure.

Number of nodes : 12

Number of links : 18

k = 2 :

Degree : 4, since inner nodes have four links.

Diameter : 7, since most distant nodes are on the k=2 triangles as pointed in the figure.

Number of nodes : 39

Number of links : 63

For the given k values, there are multiple paths between nodes which can improve the tolerance to link and node failures.

For any “k,” larger than 2 :

Degree : 4 since inner nodes have four links.

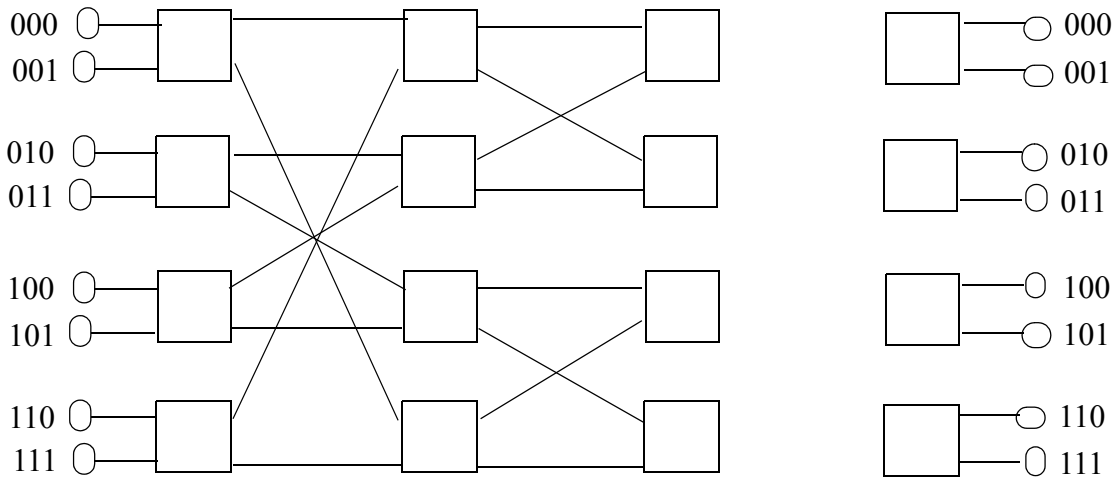
Diameter : the distance between two nodes on the far left and right or $k > 1 : (2 * k) + 3$

Number of nodes : $= \sum_{i=1}^k (3^i)$

Number of links : each time the network expands, we add $3*(k-1)*15$ links : $3 + \sum_{i=0}^k (3^i \times 15)$

Other observations : All nodes are provided with multiple paths, providing fault tolerance in case node and link failures.

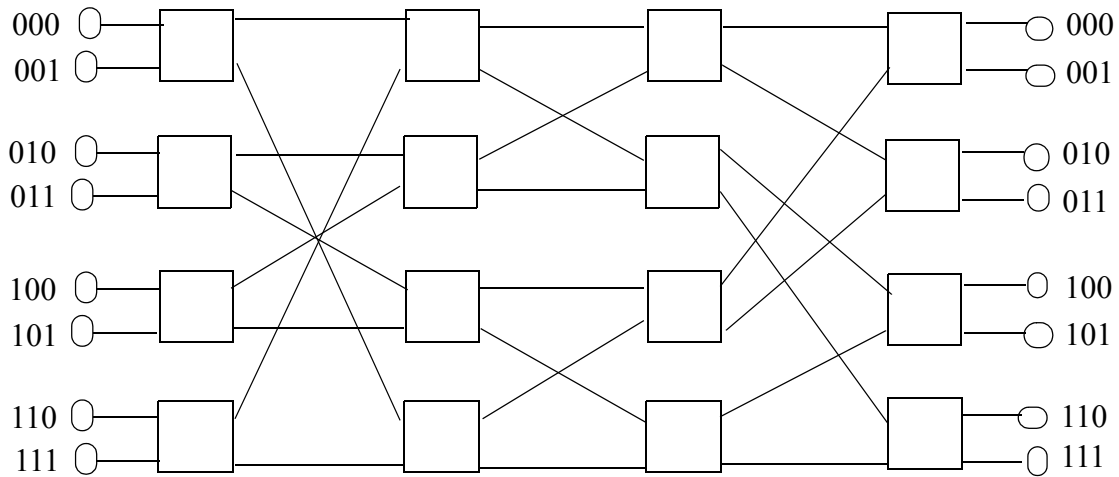
Q11) Consider the following dynamic indirect interconnection network with **eight** nodes on a side and **four** stages of 2x2 switches :



- a) Connect the rightmost two stages to each other so that
 => **Any** node on the left side can communicate with **any** node on the right side **and**
 => There are at least **two** paths between any pair of a left-side node and a right-side node

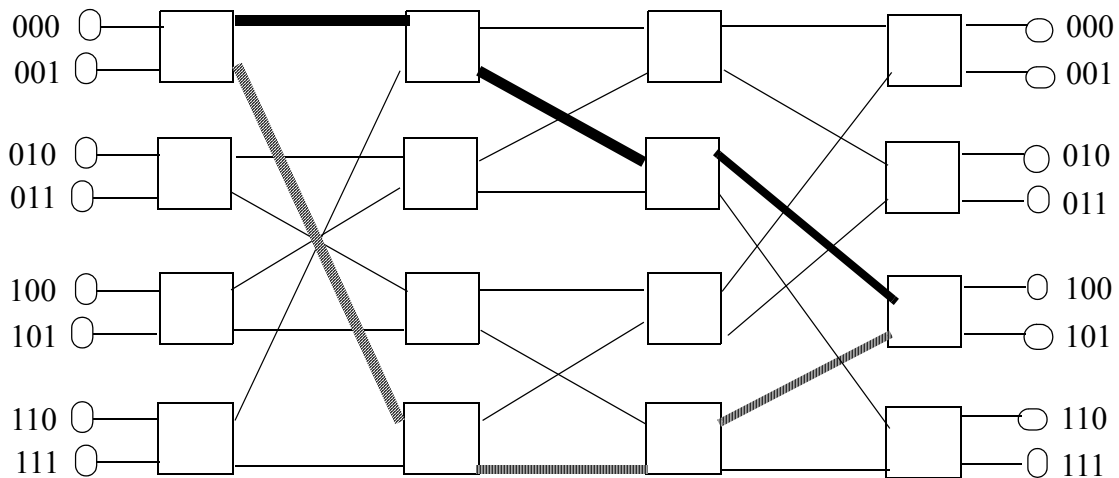
- b) Make observations on the network, including
 i) The routing. For example, show how node 001 on the left side can communicate with node 101 on the right side.
 ii) Blocking/nonblocking properties.
 iii) The interstage connection scheme.

A11) a) The interconnection network with the leftmost two stages connected :

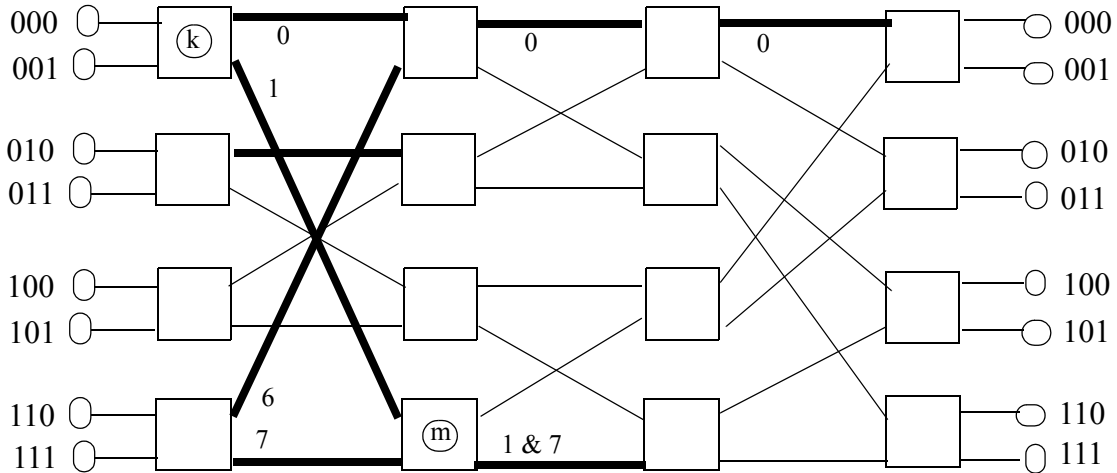


b) i) There are two paths to take for a message to reach the destination. Therefore, the routing technique used for the Omega network cannot be used since there is only one path to reach the destination in the Omega network.

The two paths from 001 to 101 are shown by thick lines in different shades :

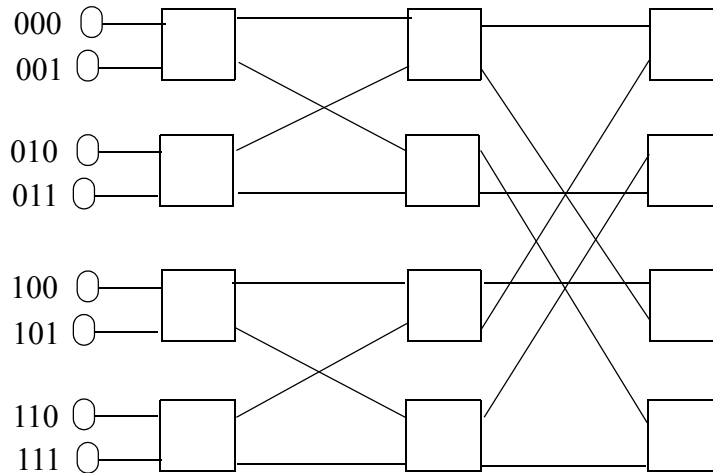


ii) The interconnection network is **blocking**. Assume that all the nodes on the left send messages to the nodes with identical id numbers on the right. That is, node 0 on the left send a message to node 0 on the right, 1 to 1, 2 to 2, 3 to 3, etc. One can see that node 0 can send a message to node 0 by using the upper links. Because of this, node 1 on the left has to use the lower link of switch with the label “k.” But, then node 1 has a conflict with node 7 in the switch labelled “m.” Both of them have to take the lower link of switch “m” at the same time which is impossible.



iii) The interstage connection mechanism between the two leftmost stages : Nodes 1 and 6 are connected by flipping their id numbers and similarly nodes 3 and 4. The other nodes are connected to the ports with identical numbers. The scheme in the middle is similar. In this case, nodes 1 and 2 have their rightmost two bits flipped while 0 and 3 are directly connected to their identical-number ports. It is similar among nodes 4, 5, 6 and 7 that 5 and 6 are connected by flipping and 4 and 7 are directly connected to the identical port numbers. The rightmost stage is a perfect shuffle with 8 nodes.

Q12) Consider the following interconnection network with **eight** nodes and **three** stages of 2x2 switches :



Each link is **bidirectional** to allow communication from its left to its right and also from its right to its left at the same time.

- a)**
- i)** Is this a direct network or an indirect network ? Explain.
 - ii)** How is the number of stages of switches calculated ?

b) Make observations on the network, including

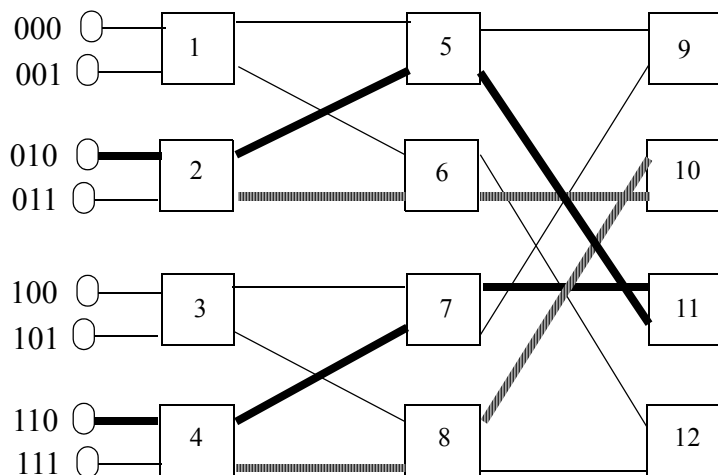
i) The routing for the above network **and** for any number of nodes. For example, show how node 010 above can communicate with node 110. Show all possible **minimal**-length paths between these two nodes.

ii) The interstage connection scheme for the above network **and** for any number of nodes.

A12) a) i) The interconnection network is an indirect network since it uses switches.

ii) The number of switches is determined by the \log_2 function. For example, if there are 8 nodes, then there are $\log_2 8 = 3$ stages of switches. In the general case where there are N nodes, the number of stages is $\log_2 N$.

b) i) There are **multiple** minimal-length paths between a pair of nodes. If two nodes have their id numbers differ rightmost position, position 0, there is $2^0 = 1$ path between them. If the most significant bit different between them is 1, then there are $2^1 = 2$ minimal-length paths between them. If the most significant bit different is 2, then there are $2^2 = 4$ minimal-length paths between them. In the case 010 - 110, the most significant bit different is 2, hence there four minimal-length paths between them. Two of the four paths from 010 to 110 are shown by thick lines in different shades :



There are four different paths from 2 to 6 :

i) 2 - 5 - 11 - 7 - 4 (Bold lines)

ii) 2 - 6 - 10 - 8 - 4 (Shaded lines)

iii) 2 - 5 - 9 - 7 - 4

iv) 2 - 6 - 12 - 8 - 4

ii) The interstage connection mechanism is **butterfly**. From left to right, the size of the butterfly connection becomes larger. For the leftmost two stages, every two rows of switches form butterflies. Next two stages have every four rows of switches forming butterflies between them, etc.