Problem 1

Illustrate the effect of round-off error in adding up numbers of differing magnitudes. Do not use a calculator or a computer, instead do everything here by hand. Assume that all arithmetical operations are rounded to 3 digits at each step. The result we want to compute is:

$$10.0 + 0.333 + 0.333 + 0.333$$

1. What is the result if you add from left to right?

2. What is the result if you add from right to left?

3. Compare relative error for the results obtained from parts 1 and 2.

Solution

1. Adding from left to right and rounding to 3 digits at each step, we have:

$$10.0 + 0.333 = 10.3$$

$$10.3 + 0.333 = 10.6$$

$$10.6 + 0.333 = 10.9$$

2. Adding from right to left and rounding to 3 digits at each step, we have:

$$0.333 + 0.333 = 0.666$$

$$0.666 + 0.333 = 0.999$$

$$0.999 + 10.0 = 11.0$$

3. The exact answer is 10.999, and so the relative error for method (1) is -9×10^{-3}, and for method (2) is -9×10^{-5}. Therefore the relative error for method (2) is about 100 times less than method (1). Roundoff error is minimized if numbers with small magnitudes are added up first.