Computing $\mathbf{A}^{-1}\mathbf{B}$

Let $\mathbf{A}$ be an n by n matrix, and $\mathbf{x}$ and $\mathbf{B}$ be both n by m matrices. Supposed matrices $\mathbf{A}$ and $\mathbf{B}$ are given, and we want to solve the linear system of equations

$$\mathbf{A}\mathbf{X} = \mathbf{B}$$

for $\mathbf{X}$. In component form this equation is

$$\sum_{j=1}^{n} A_{ij} X_{jk} = B_{ik}.$$

Therefore the problem is basically the same as the linear system

$$\mathbf{Ax} = \mathbf{b}$$

where $\mathbf{x}$ and $\mathbf{b}$ are n-vectors, except that we now have m copies of the problem, each having the same $\mathbf{A}$ but having vectors $\mathbf{x}$ and $\mathbf{b}$ taken from each of the corresponding columns of $\mathbf{X}$ and $\mathbf{B}$ respectively. Therefore LU factorization using Gaussian elimination can be used efficiently to find matrix $\mathbf{X}$ without first explicitly finding $\mathbf{A}^{-1}$ and then multiply by $\mathbf{B}$ (a very inefficient process). We only need to perform LU factorization of $\mathbf{A}$ once, then forward and backward substitution can then be done for each columns of $\mathbf{B}$ to obtain the corresponding column of $\mathbf{X}$.