Rational-Function Interpolation

Polynomials are not always the best form for representing a function or a set of data points. A rational function, which is defined as a ratio of two polynomials, is often a better choice, especially if the function to be approximated or interpolated has a pole in or near the domain of interest.

Given a set of N data points $(t_1, y_1), \cdots, (t_N, y_N)$, we seek an interpolation function of the form

$$r(t) = \frac{p_m(t)}{q_n(t)} = \frac{a_m t^m + \cdots + a_0}{b_n t^n + \cdots + b_0}$$

The rational-function interpolation is based on the Bulirsch-Stoer algorithm, which produces a "diagonal" rational function, *i.e.*, a rational function in which either $m = n$ or $m = n - 1$, depending on whether the number of data points, N, is even or odd. The algorithm is recursive and has a structure very much like in Newton's divided difference.

The following table shows the algorithm for computing the interpolated values at any specified point for the case of three data points:

(t_1, y_1)	$R_1 = y_1$		
(t_2, y_2)	$R_2 = y_2$	$R_{12} = R_2 + \dfrac{R_2 - R_1}{\dfrac{t - t_1}{t - t_2}\left[1 - \dfrac{R_2 - R_1}{R_2}\right] - 1}$	
(t_3, y_3)	$R_3 = y_3$	$R_{23} = R_3 + \dfrac{R_3 - R_2}{\dfrac{t - t_2}{t - t_3}\left[1 - \dfrac{R_3 - R_2}{R_3}\right] - 1}$	$R_{123} = R_{23} + \dfrac{R_{23} - R_{12}}{\dfrac{t - t_1}{t - t_3}\left[1 - \dfrac{R_{23} - R_{12}}{R_{23} - R_2}\right] - 1}$