ASSIGNMENT 9

Due April 20, 2004 (before start of class)

Problem 9

This is a simplified version of computer problem 8.1 on p. 378 of Heath.

Since

$$\int_0^1 \frac{4}{1+x^2} dx = \pi,$$

one can compute an approximate value for π using numerical integration of the given function.

1. Use the midpoint, trapezoid, and Simpson composite quadrature rules to compute the approximate value for π in this manner for various step size $h = (b-a)/k$, where k is the number of subintervals. Try to characterize the error as a function of h for each rule, and also compare the accuracy of the rules with each other (based on the known value of π). The vectorized versions of the midpoint, trapezoid, and Simpson composite quadrature rules that I posed on the course website use a great deal of memory especially when k is larger than 2^{20}.

   Make sure that you do not exceed that limit, otherwise MATLAB will be using virtual memory (using your hard-drive as RAM) and computation will proceed extremely slowly. When that happens you will see your hard-drive spinning nonstop for an extremely long time. You may then need to press control-alternate-delete in order to terminate the MATLAB session and to regain control of your computer.

   In the case of the Simpson composite quadrature rule, is there any point beyond which decreasing h yields no further improvement? Because of the RAM problem, you will need a lot of RAM before you can observe the same effect for the midpoint and trapezoid rules. (So do not attempt to do that using my programs unless you have more than 512Mb of RAM.)

2. Compute the above integral using several iterations of Romberg integration. Comment on your results.

Submit a hardcopy of your work and results. Do not submit an electronic copy of your Matlab program.