ASSIGNMENT 9 - Solution

Problem 9 - Solution

Please take a look at the MATLAB program for this homework problem. You need to define and use the appropriate function for the integrand. You can define the function either inline or write a separate function for it. Make sure you integrate over the correct interval. The exact result for this integral is π, so one can compute the computational errors.

The results are summarized in the following table:

h	Midpoint Error	Trapezoid Error	Simpson Error
5.0000e-001	2.0760e-002	-4.1593e-002	-8.2593e-003
2.5000e-001	5.2079e-003	-1.0416e-002	-2.4026e-005
1.2500e-001	1.3021e-003	-2.6042e-003	-1.5113e-007
6.2500e-002	3.2552e-004	-6.5104e-004	-2.3650e-009
3.1250e-002	8.1380e-005	-1.6276e-004	-3.6957e-011
1.5625e-002	2.0345e-005	-4.0690e-005	-5.7687e-013
7.8125e-003	5.0863e-006	-1.0173e-005	-1.0214e-014
3.9063e-003	1.2716e-006	-2.5431e-006	4.4409e-016
1.9531e-003	3.1789e-007	-6.3578e-007	-1.3323e-015
9.7656e-004	7.9473e-008	-1.5895e-007	8.8818e-016
4.8828e-004	1.9868e-008	-3.9736e-008	-8.8818e-016
2.4414e-004	4.9671e-009	-9.9341e-009	-2.6645e-015
1.2207e-004	1.2418e-009	-2.4835e-009	-2.6645e-015
6.1035e-005	3.1043e-010	-6.2089e-010	2.2204e-015
3.0518e-005	7.7612e-011	-1.5522e-010	-9.7700e-015
1.5259e-005	1.9384e-011	-3.8820e-011	0.0000e+000
7.6294e-006	4.8401e-012	-9.6736e-012	-1.7764e-014
3.8147e-006	1.2803e-012	-2.3936e-012	2.2204e-015
1.9073e-006	3.1664e-013	-6.2395e-013	5.5511e-014

```
IRomberg =
  Columns 1 through 3
    3.000000000000000e+000                          0                          0
    3.100000000000000e+000    3.133333333333333e+000                          0
    3.131176470588235e+000    3.141568627450981e+000    3.142117647058824e+000
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   3.138988494491089e+000    3.141592502458707e+000    3.141594094125888e+000
   3.140941612041389e+000    3.141592651224822e+000    3.141592661142563e+000
   3.141429893174975e+000    3.141592653552837e+000    3.141592653708038e+000
   3.141551963485654e+000    3.141592653589214e+000    3.141592653591639e+000
   3.141582481063753e+000    3.141592653589786e+000    3.141592653589824e+000
  Columns 4 through 6
                        0                         0                         0
                        0                         0                         0
                        0                         0                         0
   3.141585783761874e+000                         0                         0
   3.141592638396796e+000    3.141592665277717e+000                         0
   3.141592653590029e+000    3.141592653649611e+000    3.141592653638244e+000
   3.141592653589791e+000    3.141592653589790e+000    3.141592653589731e+000
   3.141592653589795e+000    3.141592653589795e+000    3.141592653589795e+000
  Columns 7 through 8
                        0                         0
                        0                         0
                        0                         0
                        0                         0
                        0                         0
                        0                         0
   3.141592653589720e+000                         0
   3.141592653589795e+000    3.141592653589795e+000

errorRomb =
   1.776356839400251e-015
```

Both the composite midpoint and trapezoid rules are of order h^2. Therefore if h is reduced by a half then the error is expected to be reduced by a fourth. The above results fully support this mathematical fact.

On the other hand, the composite Simpson's rule has order h^4. Thus if h is reduced by a half then the error is supposed to be reduced by a factor of $16(= 2^3)$. However we see that for this integral, the error is actually reduced by a factor of $64(= 2^5)$. There seems to be some unanticipated cancelling of errors for this integral. In fact if you change the integral to a different one, then you will find the error in line with the mathematical estimate.

For $h \approx 4 \times 10^{-4}$, we find that there is no further improvement to the accuracy of the result computed using Simpson's rule. Numerical round-off effects begin to contribute to and dominate the error. You can see that the error begins to change it sign.

We find that Romberg's quadrature gives the best result if we perform 7 iterations. The error is about 2×10^{-15}.