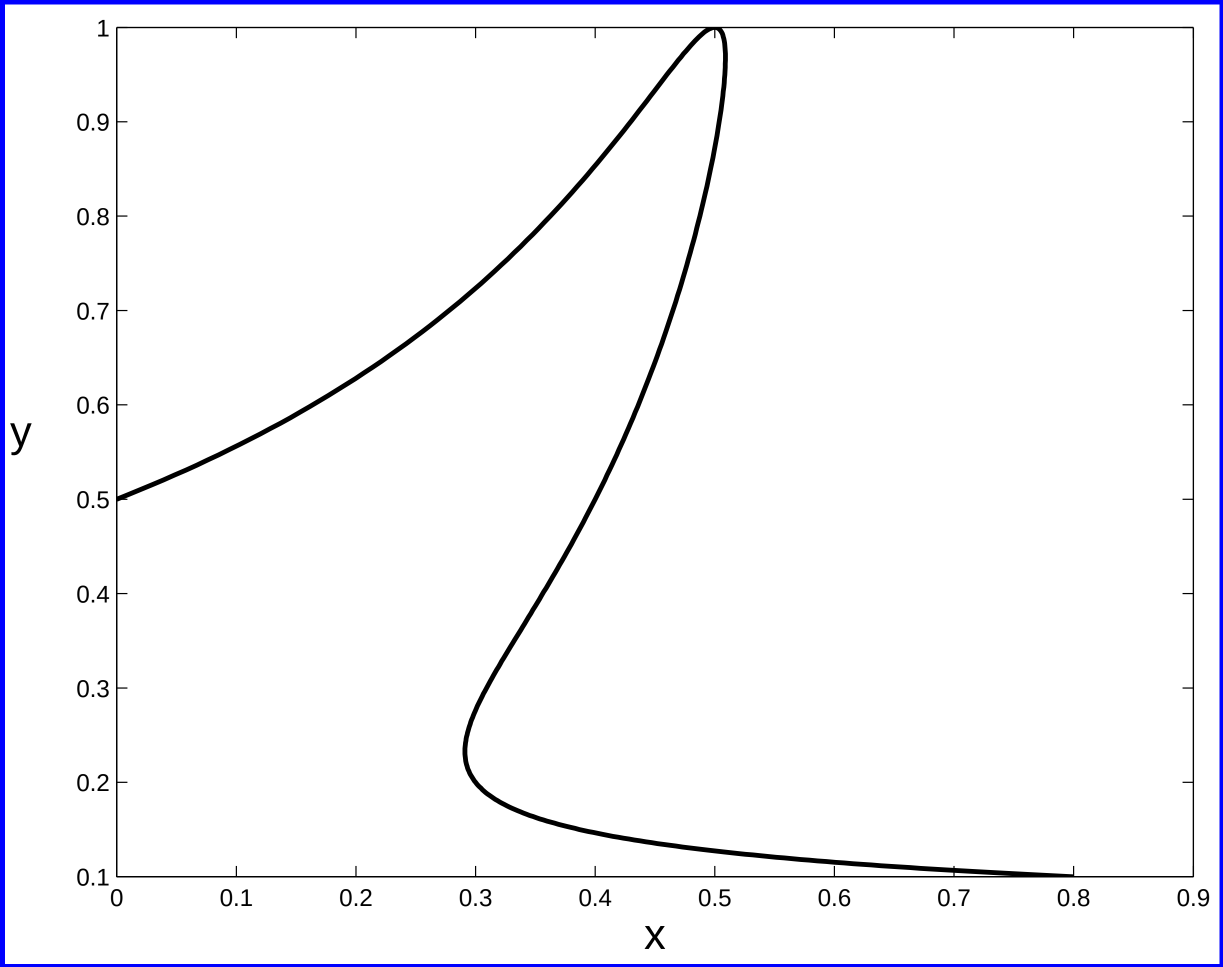
Interpolation of Parametric Curves using Cubic Spline

The curve as shown here cannot be expressed as a function of one coordinate variable in terms of the other. Therefore none of the techniques we have developed can be used to interpolate curves of this general form.


1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
y
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
x

A good mathematical treatment is to describe such a curve parametrically by a parameter t on some interval $[t_0, t_N]$. There must be a pair of functions $x(t)$ and $y(t)$ so that the curve is given by $(x(t), y(t))$ as t varies from t_0 to t_N.

The interpolation problem associated with these parametric curves can be handled as follows. Supposed that we are given $N+1$ data points:

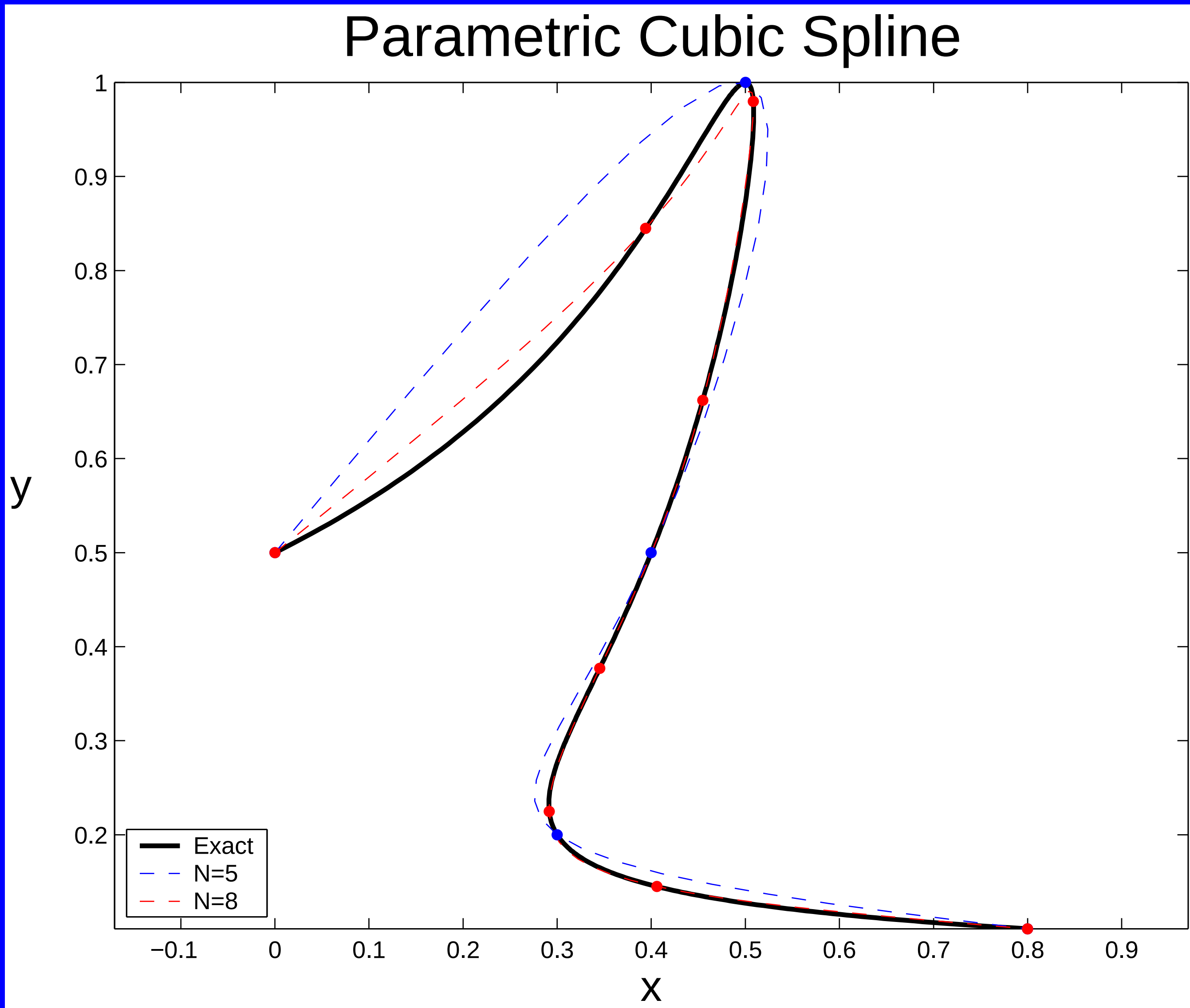
i	0	1	$\cdots$	$N-1$
t_i	t_0	t_1	$\cdots$	t_N
x_i	x_0	x_1	$\cdots$	x_N
y_i	y_0	y_1	$\cdots$	y_N

then we want to find interpolating functions $x(t)$ and $y(t)$ so that

$$x(t_i) = x_i. \qquad y(t_i) = y_i, \qquad i = 0, \cdots, N-1.$$

Thus we have to interpolate the data points (x_i, t_i) for $i = 0, \cdots, N-1$, and also the data points (y_i, t_i) for $i = 0, \cdots, N-1$.

For the curve given above, our result using cubic spline interpolation is shown in the following figure.



Parametric Cubic Spline
Exact
N=5
N=8
x
y
−0.1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1