CS3734 MIDTERM EXAMINATION, Spring 2005

1. This is an open-book examination. Only the official textbook (or hard-copies of the book), my lecture notes and homework solutions are allowed.
2. No calculators or computers of any kind allowed.
3. Put all your answers in the blue-book(s).
4. Please note that the following problems have different weights.

Problem 1 [30 pts]

What does each of the following three programs do? How many lines of output does each program produce? What are the last two values of x printed? (No need to give numerical values. An answer such as $3^{-1.98}$ is fine.)

```
% program 1
x = 1;
while 1+x > 1
   x = x/2;
   disp(x);
end

% program 2
x = 1;
while x+x > x
   x = 2*x;
   disp(x);
end

% program 3
x = 1;
while x+x > x
   x = x/2;
   disp(x);
end
```

Problem 2 [50 pts]

1. Use Gaussian elimination without pivoting to solve the linear system

   $$\mathbf{Ax} = \mathbf{b},$$

   where

   $$\mathbf{A} = \begin{bmatrix} \epsilon & 1 \\ 1 & 1 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} 1+\epsilon \\ 2 \end{bmatrix} \qquad \text{and} \qquad 0 \le \epsilon \le \epsilon_{\text{mach}}/4.$$

   Give the multiplier and matrices $\mathbf{L}$ and $\mathbf{U}$ in terms of ϵ. Show how the solution is obtained from $\mathbf{L}$ and $\mathbf{U}$.

2. Repeat part 1 using Gaussian elimination with partial row pivoting. Explain the differences with the results obtain in part 1.

Problem 3 [20 pts]

Let $\mathbf{x}$ be the solution to the linear least squares problem $\mathbf{Ax} = \mathbf{b}$, where

$$\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}$$

.

Let $\mathbf{r} = \mathbf{b} - \mathbf{Ax}$ be the corresponding residual vector. Which of the following three vectors is a possible value for $\mathbf{r}$? Why?

$$\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \quad \begin{bmatrix} -1 \\ -1 \\ 1 \\ 1 \end{bmatrix}, \quad \begin{bmatrix} -1 \\ 1 \\ 1 \\ -1 \end{bmatrix}$$

. *Hint: There is no need to actually find the solution* $\mathbf{x}$ *in order to answer this question. So we do not need to specify* $\mathbf{b}$ *either.*