Problem 1 [30 pts]

What does each of the following three programs do? How many lines of output does each program produce? What are the last two values of x printed? (No need to give numerical values. An answer such as $3^{-1.98}$ is fine.)

```
% program 1
x = 1;
while 1+x > 1
    x = x/2;
    disp(x);
end

% program 2
x = 1;
while x+x > x
    x = 2*x;
    disp(x);
end

% program 3
x = 1;
while x+x > x
    x = x/2;
    disp(x);
end
```
Problem 2 [50 pts]

1. Use Gaussian elimination without pivoting to solve the linear system

\[ Ax = b, \]

where

\[ A = \begin{bmatrix} \epsilon & 1 \\ 1 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 1 + \epsilon \\ 2 \end{bmatrix} \quad \text{and} \quad 0 \leq \epsilon \leq \epsilon_{\text{mach}}/4. \]

Give the multiplier and matrices \( L \) and \( U \) in terms of \( \epsilon \). Show how the solution is obtained from \( L \) and \( U \).

2. Repeat part 1 using Gaussian elimination with partial row pivoting. Explain the differences with the results obtain in part 1.

Problem 3 [20 pts]

Let \( x \) be the solution to the linear least squares problem \( Ax = b \), where

\[ A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}. \]

Let \( r = b - Ax \) be the corresponding residual vector. Which of the following three vectors is a possible value for \( r \)? Why?

\[ \begin{bmatrix} 1 \\ -1 \\ 1 \\ 1 \end{bmatrix}, \quad \begin{bmatrix} -1 \\ -1 \\ 1 \\ 1 \end{bmatrix}, \quad \begin{bmatrix} -1 \\ 1 \\ 1 \\ -1 \end{bmatrix}. \]

*Hint: There is no need to actually find the solution \( x \) in order to answer this question. So we do not need to specify \( b \) either.*