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The Global Positioning System

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Abstract: A brief introduction to the Global Positioning System is presented.

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The Global Positioning System (GPS) is a satellite-based navigation system made up of a network of 24 satellites placed into orbit by the U.S. Department of Defense. GPS was originally intended for military applications, but in the 1980s, the government made the system available for civilian use. GPS works in any weather conditions, anywhere in the world, 24 hours a day. There are no subscription fees or setup charges to use GPS.

1. How it works

GPS satellites circle the earth twice a day in a very precise orbit and transmit signal information to earth. GPS receivers take this information and use triangulation to calculate the user's exact location. Essentially, the GPS receiver compares the time a signal was transmitted by a satellite with the time it was received. The time difference tells the GPS receiver how far away the satellite is. Now, with distance measurements from a few more satellites, the receiver can determine the user's position.

A GPS receiver must be locked on to the signal of at least three satellites to calculate a 2D position (latitude and longitude) and track movement. With four or more satellites in view, the receiver can determine the user's 3D position (latitude, longitude and altitude). Once the user's position has been determined, the GPS unit can calculate other information, such as speed, bearing, track, trip distance, distance to destination, sunrise and sunset time and more.

2. How accurate is GPS?

Today's GPS receivers are extremely accurate, thanks to their parallel multi-channel channel receivers which can quickly lock onto satellites when first turned on and then maintain strong locks, even in dense foliage or urban settings with tall buildings. Certain atmospheric factors and other natural and man-made sources of error can affect the accuracy of GPS receivers. GPS receivers are accurate to within 15 meters on average.

Newer receivers with WAAS (Wide Area Augmentation System) capability can improve accuracy to less than three meters on average. Users can also get better accuracy with Differential GPS (DGPS), which corrects GPS signals to within an average of three to five meters. The U.S. Coast Guard operates the most common DGPS correction service. This system consists of a network of towers that receive GPS signals and transmit a corrected signal by beacon transmitters. In order to get the corrected signal, users must have a differential beacon receiver and beacon antenna in addition to their GPS.

3. The GPS satellite system

The 24 satellites that make up the GPS space segment are orbiting the earth about 27 km (12,000 miles) above us. They are constantly moving, making two complete orbits in less than 24 hours. These satellites are travelling at speeds of roughly 11 km or 7,000 miles an hour.

GPS satellites are powered by solar energy. They have backup batteries onboard to keep them running in the event of a solar eclipse, when there's no solar power. Small rocket boosters on each satellite keep them flying in the correct path.

Here are some other interesting facts about the GPS satellites:

- The first GPS satellite was launched in 1978.
- A full constellation of 24 satellites was achieved in 1994.
- Each satellite is built to last about 10 years.
- Replacements are constantly being built and launched into orbit.
- A GPS satellite weighs approximately 2,000 pounds and is about 17 feet across with the solar panels extended.
- Transmitter power is only 50 watts or less.

4. What's the signal?

GPS satellites transmit two low power radio signals, designated L1 and L2. Civilian GPS uses the L1 frequency of 1575.42 MHz in the UHF band. The signals travel by line of sight, meaning they will pass through the air and clouds but will not go through most solid objects such as buildings and mountains.

A GPS signal contains three different bits of information a pseudorandom code, ephemeris data and almanac data.

- The pseudorandom code is simply an I.D. code that identifies which satellite is transmitting information.
- Ephemeris data, which is constantly transmitted by each satellite, contains important information about the status of the satellite (healthy or unhealthy), current date and time. This part of the signal is essential for determining a position.
- The almanac data tells the GPS receiver where each GPS satellite should be at any time throughout the day. Each satellite transmits almanac data showing the orbital information for that satellite and for every other satellite in the system.

5. Sources of GPS signal errors

Factors that can degrade the GPS signal and thus affect accuracy include the following:

- Ionosphere and troposphere delays The satellite signal slows as it passes through the atmosphere. The GPS system uses a built-in model that calculates an average amount of delay to partially correct for this type of error.

- Signal multipath This occurs when the GPS signal is reflected off objects such as tall buildings or large rock surfaces before it reaches the receiver. This increases the travel time of the signal, thereby causing errors.
- Receiver clock errors A receiver's built-in clock is not as accurate as the atomic clocks onboard the GPS satellites. Therefore, it may have very slight timing errors.
- Orbital errors Also known as ephemeris errors, these are inaccuracies of the satellite's reported location.
- Number of satellites visible The more satellites a GPS receiver can "see", the better the accuracy.
- Buildings, terrain, electronic interference, or sometimes even dense foliage can block signal reception, causing position errors or possibly no position reading at all. GPS units typically will not work indoors, underwater or underground.
- Satellite geometry/shading This refers to the relative position of the satellites at any given time. Ideal satellite geometry exists

when the satellites are located at wide angles relative to each other. Poor geometry results when the satellites are located in a line or in a tight grouping. The resulting mathematical problem becomes higher ill-conditioned.

- Intentional degradation of the satellite signal Selective Availability (SA) is an intentional degradation of the signal once imposed by the U.S. Department of Defense. SA was intended to prevent military adversaries from using the highly accurate GPS signals. The government turned off SA in May 2000, which significantly improved the accuracy of civilian GPS receivers.

6. The Mathematics Related to GPS

A given satellite k broadcasts its current position (x_k, y_k, z_k) and the current time t_k. The GPS receiver records this information from the the satellite along with the time, τ_k, of receipt of the message. However the clock in the receiver is much less accurate compared with the clock onboard the satellite, and so it is expected to be off by a certain amount, t. The actual time of receipt of the message is therefore not

τ_k but $\tau_k - t$. If t is positive, then the clock is t seconds too fast, and if t is negative, then the clock is t seconds too slow. Therefore the signal takes $(\tau_k - t) - t_k$ seconds to arrive at the receiver from the satellite, and so the receiver must be located some where on the surface of a sphere of radius $c(\tau_k - t_k - t)$ centered at (x_k, y_k, z_k). Here c is the speed of light which has the value 2.99792458 m/s exactly in vacuum.

If the receiver is located at (x, y, z), then we have the relation

$$(x - x_k)^2 + (y - y_k)^2 + (z - z_k)^2 = c^2(d_k - t)^2,$$

where we define the apparent transit time $d_k = \tau_k - t_k$. Since typically $|d_k|$ is much smaller than $|t|$, high precision arithmetics is required for the receiver. Because the position of the receiver (x, y, z) and the receiver clock error t are unknown, we need the data (x_k, y_k, z_k) and t_k from at least 4 satellites. Assuming that we have such information from 4 satellites, then the unknowns x, y, z, t can be determined from the roots of the following system of 4 nonlinear equations,

$$f_k(x, y, z, t) = (x - x_k)^2 + (y - y_k)^2 + (z - z_k)^2 - c^2(d_k - t)^2, \quad k = 1, 2, 3, 4.$$

It is important to note that t, the error of the receiver clock, must be independent of k, *i.e.* the same for each satellite.

The receiver solves the system typically using nonlinear least squares, since it is better to receive data from more than satellites, so that the unknowns can be determined more accurately. Here we will assume that there is data precisely from 4 satellites and find the roots using Newton's method for multidimensional systems. The Jacobian matrix can be computed analytically to give

$$\mathbf{J}(x,y,z,t) = 2 \begin{bmatrix} x-x_1 & y-y_1 & z-z_1 & -c^2(d_1-t) \\ x-x_2 & y-y_2 & z-z_2 & -c^2(d_2-t) \\ x-x_3 & y-y_3 & z-z_3 & -c^2(d_3-t) \\ x-x_4 & y-y_4 & z-z_4 & -c^2(d_4-t) \end{bmatrix}$$

Consider a typical set of data given in the following table.

x_k $(\times 10^6\ m)$	y_k $(\times 10^6\ m)$	z_k $(\times 10^6\ m)$	d_k $(\times 10^{-9}\ s)$
1.876371950559744	-10.64143413406656	24.26976465661440	0.07234683200
10.97666464137408	-13.08142752230029	20.35116937827073	0.06730845726
24.58513954435968	-4.335023426659201	9.086300320217470	0.06738499643
3.854136195752833	7.248575943442946	25.26630462778753	0.07651971839

We find that

$$x = 4.576010710428177 \times 10^6 m$$

$$y = -2.318239699207955 \times 10^6 m$$
$$z = 3.770361173258234 \times 10^6 m$$

and $t = -2.000493757326701 \times 10^{-3} s$.

As you can see, the error in the receiver clock, $|t|$, is indeed several orders of magnitude larger than the signal transit times, d_k.

Please consult the references below if you want to learn more about the subject.

References

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