ASSIGNMENT 1

Due September 15, 2005, before 11:00 am

Problem 1

A transportation engineer in charge of a project to re-surface a portion of a multi-lane highway must reduce the traffic to a single lane during construction. The goal is to find and post an optimal speed that will minimize traffic congestion and therefore maximize the traffic flow rate along the road.

Here is the information the engineer needs in order to determine the optimal speed.

- Assume that all the cars have length ℓ, travel at speed v, and are spaced by a separation distance s measured from the front of a car to the back of the car directly ahead.
- Cars need to keep a safe distance between them. The faster they travel, the larger the distance needs to be. Specifically, the minimum safe separation is

  $$s = t_r v + \frac{v^2}{2\mu g},$$

  where t_r is the driver's reaction time, μ is the coefficient of friction between tires and road, g is the acceleration due to gravity.
- To minimize the congestion, one needs to maximize the traffic flow rate along the road. The number of cars passing a given point on the road per unit time, r, is the best measure of the flow rate.

First we need to find the number of cars passing a given point on the road per unit time, r. Since the distance traveled by a car in time t is vt, and there is one car on each segment of length $\ell + d$ of the road, the number of cars on a strip of the road of length vt is therefore given by $vt/(\ell + d)$. Dividing this number by t, we have the flow rate

$$r = \frac{v}{\ell + t_r v + \dfrac{v^2}{2\mu g}}$$

1. The best way before working on any mathematical equation is to have it reduced in dimensional form. In general that can be accomplished in more than one way.

2. Then assume that $\ell = 2\ m$, $\mu = 0.6$, $g = 9.8\ m/s^2$, and $t_r = 0.3\ s$, plot the flow rate as a function of the car speed in dimensionless units.

3. Then use the Golden Section Search method to find the optimal car speed v^* that maximizes the flowing rate.

4. Calculate also the resulting flow rate r^*.

5. Repeat you work using different sets of data for ℓ, μ, g, and t_r.

6. What do you find for v^* and r^* (in both actual values and in dimensionless units)?