

ASSIGNMENT 6

Due December 1, 2005, before 11:00 am

Problem 6

We want to consider the integrals of the functions

$$f(x; k) = x^k,$$

from 0 to 1 for $k = 0, 1, \dots$.

1. This family of integrals can be done easily by hand. What is the solution as a function of k ?
2. Compute this family of integrals for $k = 0, 1, \dots$ numerically using a 4-point Gauss-Legendre Quadrature rule. Two of the roots are

$$\frac{\sqrt{525 - 70\sqrt{30}}}{35} \quad \frac{\sqrt{525 + 70\sqrt{30}}}{35},$$

and their corresponding weights are

$$\frac{(18 + \sqrt{30})}{36} \quad \frac{(18 - \sqrt{30})}{36}.$$

Recall that for any even-point Gauss-Legendre Quadrature rule, the roots are even and for each positive root there is a root of opposite sign, and they both have the same weights. Plot the absolute value of the relative error as a function of k . Explain your finding in detail.