ASSIGNMENT 3

March 9, 2004 (before 6:00 pm)

Homework 4: Evaluating a 2-Dimensional Integral Using the Sample-Mean Monte Carlo Method

Use the Sample-Mean Monte Carlo method to evaluate the following 2-dimensional integral:

$$I = \int_0^{\pi/2} \int_0^{\pi/2} \frac{\sin x_2 \sqrt{1 - k^2 \sin^2 x_1 \sin^2 x_2}}{1 - k^2 \sin^2 x_2} dx_1 dx_2.$$

where $k = 3/5$. The exact answer turns out to be $5\pi/8$.

1. Use $100,000$ points in your simulation to obtain a value for I. Also obtain the most probable error which you can estimate from the simulation data. You need to repeat the calculation a number (say 20) of times to obtain an average value. Compare the error for that average value with the most probably error.

2. Next investigate the dependence of the error with the number of points used in the simulation. Use $N = 100, 1,000, 10,000$, and $100,000$. For each value of N repeat the calculation 20 times and compute the average of the errors. Plot the logarithm of the average errors against the negative of the logarithm of N. How does the average errors depend on N?