Homework Assignment 2

P1: Minimize \( z \)

\[
\begin{align*}
z &= x_1 + x_2 + x_4 \\
x_1 &= x_2 + x_3 + x_5 = 1 \\
x_2 &= -x_3 + x_4 + x_6 = 5 \\
x_1 &= -3x_2 + x_3 + x_4 + x_7 = 4 \\
x_1 &\geq 0 \quad x_2 \geq 0 \quad x_3 \geq 0 \quad x_4 \geq 0 \quad x_5 \geq 0 \quad x_6 \geq 0 \quad x_7 \geq 0
\end{align*}
\]

1. Show that \( x^* \) given by \( x_5^* = 1, x_6^* = 5, x_7^* = 4, \) and \( x_1^* = x_2^* = x_3^* = x_4^* = 0 \) is an extreme point (definition on p. 2 of Appendix) of

\[
D_1 = \left\{ x = (x_1, x_2, \ldots, x_7) \mid x_1 + x_2 + x_3 + x_5 = 1; x_2 - x_3 + x_4 + x_6 = 5; x_1 - 3x_2 + x_3 + x_4 + x_7 = 4; x_j \geq 0, j = 1,\ldots, 7 \right\}
\]

2. Show that \( D_2 \) is a cone; characterize the cone.

\[
D_2 = \left\{ x = (x_1, x_2, \ldots, x_7) \mid x_1 + x_2 - x_3 + x_5 = 1; x_2 - x_3 + x_4 + x_6 = 5; x_1 - 3x_2 - x_3 + x_4 + x_7 = 4; x_j \geq 0, j = 1,\ldots, 7 \right\}
\]

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