HOMEWORK I

DUE : September 21, 2004

READ : Related portions of Chapter I and Chapter II

ASSIGNMENT : There are seven questions four of which are from Chapters I and II of the textbook

Solve all homework and exam problems as shown in class and past exam solutions

1) Solve Problem 1.5.

Can you think of an analog system that may stay as analog for some time to come in the future?

2) Solve Problem 2.5 (c) and (e).

Assume that all binary numbers are unsigned binary numbers. Show manual calculations, indicating that you did not use a calculator.

3) Solve Problem 2.10 (b) and (c).

First, convert the hex digits to bit patterns. Second, by assuming that these bit patterns represent 2’s complement numbers, perform the additions in binary during which show all carries. State if there is an overflow and why.

4) Solve Problem 2.16 (f).

The question asks about the radix of the following operation:

\[ \sqrt{(41)_{2}} = (5)_{2} \]

Clearly describe how you arrive at the result.

Hint : try to make use of the general conversion formula.

5) Perform the following operation in 2’s complement arithmetic, by using 12 bits per number:

\[(192)_{10} + (F08)_{\text{Hex}} = (?)_{10}\]

You will show all number conversions as done in class. Note that the Hex digits are for coding purposes. They represent a 2’s Complement number. Make observations on the addition.
6) Convert the following fixed-point decimal number to a 16-bit 2’s complement fixed-point binary number:

\[(525.3125)_{10}\]

Use four (4) bits for the fraction part of the 2’s complement number (thus 12 bits for the integer part).

7) Calculate the following logarithm:

\[\log_2[(010000)_{2\text{ 2’s Complement}}] = (?)_{10}\]

RELEVANT QUESTIONS AND ANSWERS

Q1) Convert the following decimal number to a 16-bit 2’s complement binary number, by using 7 bits for the integer portion: \((-26.75)_{10}\)

A1) First, we have to consider \((+26.75)_{10}\) since we cannot directly convert negative numbers. For the binary number, the integer part is obtained by successive divisions and the fraction part is obtained by successive multiplications:

\[
\begin{align*}
\text{The integer part:} & & \\
26/2 &= 13 \quad \text{&} \quad 0 \quad \text{(lsb)} \\
13/2 &= 6 \quad \text{&} \quad 1 \\
6/2 &= 3 \quad \text{&} \quad 0 \\
3/2 &= 1 \quad \text{&} \quad 1 \\
1/2 &= 0 \quad \text{&} \quad 1 \quad \text{(msb)} \\
\end{align*}
\]

\[
(+26)_{10} \xrightarrow{} (11010)_2
\]

\[
\begin{align*}
\text{The fraction part:} & & \\
0.75 \times 2 &= 1.5 \quad \text{\rightarrow} \quad 1 \\
0.5 \times 2 &= 1.0 \quad \text{\rightarrow} \quad 1 \\
\end{align*}
\]

\[
(.75)_{10} \xrightarrow{} (.11)_2
\]

\[
(+26)_{10} \text{ using 7 bits requires sign extension:} \quad (0011010)_2
\]

\[
(-26.75)_{10} = (-0011010.110000000)_{2} = (1100101.010000000)_2
\]
Q2) Perform the following operation in 2’s complement arithmetic. The numbers are shown in the Hexadecimal notation. Thus, first convert the numbers to binary, and then add them. Make observations on the addition:

\[
\begin{array}{c}
F 6 \\
+ 4 9 \\
\hline
? 
\end{array}
\]

A2) By replacing each hexadecimal digit with four bits we convert them to 2’s Complement numbers:

\[
\begin{array}{c}
F 6 \rightarrow 1 1 1 1 \ 0 1 1 0 \\
4 9 \rightarrow 0 1 0 0 \ 1 0 0 1 \\
\hline
1 1 1 1 \ 0 1 1 0 \ + \ 0 1 0 0 \ 1 0 0 1 \\
\hline
1 1 1 1 \ 1 1 1 1 \ c_{\text{out}} 1 \rightarrow (3 \ F)_{\text{Hex}}
\end{array}
\]

There is no overflow, since the two numbers added have different sign bits: one is negative and the other is positive. Thus, the result cannot exceed the limits for 8-bit 2’s complement numbers: (-128)\(_{10}\) and (+127)\(_{10}\). The c\(_{\text{out}}\) bit is the carry out from the leftmost bit position and it is 1.

Q3) Hex digits are used to represent two numbers that are 16-bit 2’s complement numbers:

4AF8 \(-\) 1B5E \(=\) (?)\(_{10}\)

Perform the subtraction operation, by converting it to a 16-bit addition operation. Show the result in decimal.

A3) First, we convert the digits to bit strings:

\[
\begin{array}{c}
0100 \ 1010 \ 1111 \ 1000 \ \text{by using 16 bits} \\
4 \ A \ F \ 8
\end{array}
\quad
\begin{array}{c}
0001 \ 1011 \ 0101 \ 1110 \ \text{by using 16 bits} \\
1 \ B \ 5 \ E
\end{array}
\]

In order to convert the subtraction to an addition, we need to take the 2’s complement of the second number:

\[
(0001 \ 1011 \ 0101 \ 1110)_{2} = (1110 \ 0100 \ 1010 \ 0010)
\]

\[
\begin{array}{c}
0 1 0 0 \ 1 0 1 0 \ 1 1 1 1 \ 1 1 0 0 \\
+ \ 0 0 1 0 \ 1 1 1 1 \ 1 0 0 1 \ 1 0 1 0 \\
\hline
1 0 0 1 0 1 1 1 1 0 0 1 0 1 0 0 0 1 0 1 0 1 0 1 0
\end{array}
\]

The corresponding decimal number is:

0010 1111 1001 1010

15 14 13 12 11 10 9 8 7 6 5 4 3 2 1 0
By numbering the bit positions from right to left, starting at 0, we convert the binary number to a decimal number:

$$2^1 + 2^3 + 2^4 + 2^7 + 2^8 + 2^{10} + 2^{11} + 2^{13} =$$

$$= 2 + 8 + 16 + 128 + 256 + 512 + 1024 + 2048 + 8192$$

$$= (12186)_{10}$$

**Observation**: we added a negative number and a positive number, therefore, there cannot be any overflow.

**Q4)** Perform the following subtraction operation on the two 8-bit 2’s complement binary numbers below, by converting it to an 8-bit addition operation. Also, determine the missing bits and show the result in decimal:

$$a = \begin{array}{ccccccccc} \text{101111000} \\ \text{-} \begin{array}{ccccccccc} \text{1??0011} \\ \text{s} \end{array} \end{array}$$

$$= \begin{array}{ccccccccc} \text{?101??} \end{array}$$

$$= (?)_{10}$$

**A4)**

$$\begin{array}{ccccccccc} \text{10111000} \\ \text{-} \begin{array}{ccccccccc} \text{1??0011} \end{array} \end{array}$$

$$= \begin{array}{ccccccccc} \text{?101??} \end{array}$$

We need to take the 2’s complement of (b) to convert the subtraction to an addition first:

$$\begin{array}{ccccccccc} \text{(b)} = (1??0011) \end{array}$$

$$= (b)$$

$$\begin{array}{ccccccccc} \text{10111000} \\ \text{+} \begin{array}{ccccccccc} \text{0011} \end{array} \end{array}$$

$$= \begin{array}{ccccccccc} \text{11010101} \end{array}$$

The sum is a negative number. We cannot convert it to decimal directly. We will convert the negative of the sum to decimal:

$$(-\text{sum}) = (11010101) = 00101011 = 2^0 + 2^1 + 2^3 + 2^5 = 1 + 2 + 8 + 32 = (+43)_{10}$$. The sum is (-43)$_{10}$.

The number with unknown bits (number b) is a negative number, its negative (-b) is obtained above and then added:
(-b) = 0 0 0 1  1 1 0 1  
\( = 2^0 + 2^2 + 2^3 + 2^4 = 1 + 4 + 8 + 16 = (+29)_{10} \)

\[ b = 1 1 1 0 0 0 1 1 = (-29)_{10}. \]

Note that we added a negative number (-a) and a positive number (-b), therefore, there cannot be any overflow.

**Q5)** Without using a calculator, perform the following addition in *binary* as shown in class:

\[
\begin{array}{cccccccc}
0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\
+ & & & & 1 & 0 & 1 & 0 \\
\hline
? & & & & & & &
\end{array}
\]

2’s complement numbers

Make observations on the addition. Finally, convert the result to decimal.

**A5)** We are given the following addition:

\[
\begin{array}{cccccccc}
0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\
+ & & & & 1 & 0 & 1 & 0 \\
\hline
? & & & & & & &
\end{array}
\]

2’s complement numbers

Since it is the 2’s complement system and we see that the second number is shorter than the first, we sign extend the second number so both numbers have eight bits. The sign bit of the second number is 1, that is, it is negative, so we have to concatenate 1s to the left of it. Now we have the following addition:

\[
\begin{array}{cccccccc}
0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\
+ & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 \\
\hline
? & & & & & & &
\end{array}
\]

We perform the addition now:

\[
\begin{array}{cccccccc}
0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\
+ & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 \\
\hline
0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & c_{out} 1
\end{array}
\]

Observation: we know that there cannot be any overflow since we added a positive number (the first number) and a negative number (the second number).

We convert the result to decimal directly since the result is positive:

\[
\begin{array}{cccccccc}
0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\
7 & 6 & 5 & 4 & 3 & 2 & 1 & 0
\end{array}
\]

\[ 2^6 + 2^3 + 2^2 \rightarrow 64 + 8 + 4 \rightarrow (76)_{10} \text{ the result} \]
Q6) Calculate the minimum number of bits necessary to represent the following decimal number in the 2’s complement system:

$(92)_{10}$

A6) First, we have to use the formula discussed in class to determine the minimum number of bits needed to represent the decimal number in the unsigned binary system:

$$\left[ \log_2 (92 + 1) \right] = \left[ \log_2 93 \right] = \left[ 6.53 \right] = 7$$

In the 2’s complement system, one additional bit is needed as the sign bit. Therefore, we need at least 8 bits to represent $(92)_{10}$ in the 2’s Complement system.

**CONTACTS:**

1) Students can see the professor and teaching adjuncts (TAs), about the lectures, homework, and lab experiments.

2) Professor’s contact information:

   Room : 114 LC    (718) 260-3101    Fax : (718) 260-3609    haldun@photon.poly.edu

   Open-door policy to see the professor    If the door is closed, he might be in the lab

3) The TAs of the course are Nikhil Joshi, Sapan Shenoy, Jeff Tao, Haobo Wang, Bo Yang and Peng Yao.

4) All handout and lab files are at the course web site: [http://cis.poly.edu/cs2204](http://cis.poly.edu/cs2204)

5) When short-term problems are encountered in PC labs, students are advised to contact: help@duke.poly.edu or (718) 260-3123 or go to Room : 325 RH.

   For CS2204 lab related issues and to have the lab open, students need to contact Mr. Keni Yip at (718) 260-3023, keni@poly.edu. His office is 225RH

   For longer-term problems in PC labs and any other matter, students should not hesitate to contact the professor and TAs.