Problem 6

Write a VBA Sub procedure to accomplish the following task.

The \textbf{sine} function has the following Taylor series expansion:

\[
\sin(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots,
\]

where \( k! \) is the factorial function defined so that

\[
k! = k(k-1)\ldots3 \times 2 \times 1.
\]

The factorial function can also be defined recursively by the relations

\[
k! = k(k-1)! \quad \text{with} \quad 1! = 1 \quad \text{and} \quad 0! = 1.
\]

For example, we have

\[
3! = 3 \times 2 \times 1 = 6, \quad 5! = 5 \times 4 \times 3 \times 2 \times 1 = 120.
\]

For this problem you \textbf{must not} attempt to compute the factorial function. There is also no need to use the power operator, \(^\). Instead use a technique similar to what we used to compute the exponential function.

For each value of \( x \) input by the user, compute the corresponding value of \( \sin(x) \). Make sure that the absolute error is no bigger than the tolerance, \( \text{tol} = 1e-6 \). Keep track of the total number of terms that must be summed to achieve this level of accuracy. Your program should display your computed value together with the ”exact” value obtained by using the built-in \textbf{sin} function in VBA.

Display a table showing the values of \( x \), the value of \( \sin(x) \) computed, the ”exact” value of \( \sin(x) \) (according to MATLAB’s sine function), and the total number of terms that was used in computing the sum. There should be one row of output numbers for each value of \( x \).