Homework 5: Variance Reduction in Monte Carlo Simulation using the Rejection Method

Use Monte Carlo method to evaluate the following 1-dimensional integral:

\[ I = \int_{0}^{\pi/2} \sin(x) \, dx. \]

Reduce the variance using the rejection method with the following choice of comparison function:

\[ w(x) = \begin{cases} 
  x, & \text{if } 0 \leq x \leq 1 \\
  1, & \text{if } 1 \leq x \leq \frac{\pi}{2}. 
\end{cases} \]

1. Use 1000 sample points and repeat the simulation 10 times. Take the average value as your estimation of the integral. What is the actual error when compared with the exact theoretical value of 1?

2. Compute also the estimated value of the probable error based on your simulated result. This estimated value of the error should not vary much from one run to the next, and so there is no need to use the average value.

3. Since the exact theoretical answer is known in this problem, you should be able to derive a theoretical expression for the probable error. Compare the actual error, the estimated error and the theoretical error.